Kinetic theory of Jeans instability

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Kinetic treatment of the Jeans gravitational instability, with collisions taken into account, is presented. The initial-value problem for the distribution function which obeys the kinetic equation, with the collision integral conserving the number of particles, is solved. Dispersion relation is obtained and analyzed. New modes are found. Collisions are shown not to affect the Jeans instability criterion. Although the instability growth rate diminishes, the collisions they cannot quench the instability. However, the oscillation spectrum is modified significantly: even in the neighborhood of the threshold frequency $\omega=0$ (separating stable and unstable modes) the spectrum of oscillations can strongly depend on the collision frequency. Propagating (rather than aperiodic) modes are also found. These modes, however, are strongly damped.

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I. INTRODUCTION

In his letter to Richard Bentley, Isaac Newton (1692) first suggested that self-gravity in infinite universe would lead to the observed mass distribution [1]. Jeans (1902) gave the first quantitative description of fragmentation of matter due to self-gravity [2]. He has shown that a self-gravitating infinite uniform gas at rest should be unstable against small perturbations proportional to $\exp[i(\mathbf{k}\cdot\mathbf{r}-\omega t)]$. Linearization of the equations of ideal hydrodynamics and the Poisson equation for the gravitational potential results in the dispersion equation

$$\omega^2 = c_s^2 k^2 - \Omega^2, \tag{1}$$

where $\Omega=(4\pi G\rho)^{1/2}$ is the Jeans gravitational frequency, ρ is the density, $c_s=(\gamma T/m)^{1/2}$ is the adiabatic sound velocity, $\gamma=5/3$ is the ratio of specific heats, T is the gas temperature in energy units, m is the particle mass, and G is the gravitational constant. As seen from Eq. (1), ω^2 becomes negative, and the instability arises when the perturbation wavelength $\lambda=2\pi/k$ exceeds the critical value:

$$\lambda > \lambda_c = c_s\sqrt{\frac{\pi}{G\rho}}. \tag{2}$$

The pressure gradient tends to quench the instability. This force dominates over the gravity force resulting in stabilization with $\lambda\leqslant\lambda_c$. Thus, an originally uniform gas, due to the instability, should break into clots with characteristic size of the order of λ_c. It is worthy of mentioning that $k_c^{-1}=\lambda_c/2\pi=c_s/(4\pi G\rho)^{1/2}$ is the only specific characteristic length scale, which may be constructed of the parameters inherent for the problem under consideration.

Kinetic theory of the Jeans instability was given in Refs. [3–5] using methods of plasma physics. Following Landau [6] in Ref. [3] an initial-value problem has been solved for the collisionless Boltzmann-Vlasov equation by using the Landau bypass rule. Kinetic treatment of the Jeans instability is discussed in detail in Ref. [7]. The gravitating charged collisionless dust, in which the Jeans instability arises, was recently considered in Ref. [8].

It is generally believed (e.g., Ref. [7]) that the Jeans model suffers from basic inconsistency as the equilibrium state in this model does not obey the Poisson equation. Indeed, the Poisson equation does not hold in the equilibrium but this may hardly be considered as an inconsistency. Strictly speaking, the gravity force vanishes at any point of the infinite uniform gas due to symmetry reasons, so that the Poisson equation becomes irrelevant. Thus, Jeans's model, logically, is self-consistent, no matter whether or not such an equilibrium may exist. After all, ideal fluid do not exist either.

It is worthy of mentioning that more realistic models, like the expanding Newtonian world model [9], lead to the same Jeans instability criterion (2). In fact, very similar results have been obtained in Ref. [10] by using the Friedmann solution for expanding universe. Therefore, the kinetic effect of collisions will be studied below in the simplified Jeans model of an infinite gas at rest in the unperturbed state. The uniform gas under consideration is assumed to consist of two components with masses m and M ($m\ll M$). Naturally, for one-component gas collisions of a single species cannot change the momentum and density perturbations and spectra with $k\to 0$.

II. DISPERSION RELATION WITH COLLISIONS

The Boltzman-Vlasov equation for the distribution function of the light component $f(\mathbf{r},\mathbf{v},t)$ is

$$\frac{\partial f}{\partial t} + \mathbf{v}\cdot\nabla f - \nabla\varphi\cdot\frac{\partial f}{\partial \mathbf{v}} = I_{col}, \tag{3}$$

where φ is the gravitational potential which obeys the Poisson equation

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$$\nabla^2\varphi = 4\pi G\rho. \tag{4}$$

Here ρ is the full mass density of the matter. The collision term in the kinetic equation (3) describes the collisions due to attraction between the light masses and between light and heavy components. In general it has to be taken in the Landau form [11] with the "gravitational charges" Gm^2 and GmM. The characteristic collision frequencies can be written in the form $\nu_{mm} \simeq G^2 m^{7/2} n L_{mm}/T^{3/2}$ and $\nu_{mM} \simeq G^2 m^{3/2} M^2 N L_{mM}/T^{3/2}$, respectively, where n and N are the average number densities. The quantities L_{mm}, L_{mM} are the gravitational analogs of the Landau logarithm for the Coulomb-type interaction. On the assumption $\nu_{mm} \ll \nu_{mM}$, which can be easily fulfilled due to the inequality $m \ll M$, we can neglect collisions between the light particles. The distribution function for the heavy component can be taken Maxwellian, because the inequality $\nu_{MM} \simeq G^2 M^{7/2} N L_{MM}/T^{3/2} \gg \Omega$.

For our purposes and conditions under consideration we can use the simplified collision integral I_{col} in the form suggested by Bhatnagar, Gross, and Krook [12] (and applied for Coulomb collisions in plasma in [13]):

$$I_{col} = -\nu\left(f - f_0 \int f d^3\mathbf{v}\right). \tag{5}$$

Here f_0 and $n_0 = \rho_0/m$ are, respectively, the distribution function [$\int f_0 d\mathbf{v} = 1$ in contrast with the distribution $f(\mathbf{r},\mathbf{v},t)$ normalized to the inhomogeneous density] and the number density ($n_0 \equiv n$) at the initial moment $t=0$: $f(\mathbf{r},\mathbf{v},t=0) = n_0 f_0(\mathbf{v})$. In contrast to so-called "τ-approximation" (see, e.g., Eq. (6.7.1) in Ref. [14]), which cannot conserve the number of particles, Eq. (5) is compatible with the required particle conservation law: $\int I_{col} d^3\mathbf{v} = 0$. Using a similarity between the Coulomb and Newton gravity laws we can take the effective collision frequency $\nu \equiv \nu_{mM}$ between the light and heavy masses for a self-gravitating system as follows:

$$\nu = \frac{4\sqrt{2\pi} G^2 M^2 m^2 N}{3\mu^2 v_T^3}\Lambda. \tag{6}$$

Here $v_T = \sqrt{T/m}$ is the thermal velocity of the granular gas with the mass m, $\Lambda \equiv L_{mM}$ is the gravitational analog of the generalized Coulomb logarithm [15], and $\mu \simeq m$ is the reduced mass for the components considered. Equation (6) describes the elastic gravitational collisions. The processes of inelastic gravitational collisions can be taken into account by generalization of the collision integral. For example, for the case of the additional inelastic process, when small masses can fall on heavy ones, instead of ν describing by Eq. (6) we have

$$\nu = \frac{8\sqrt{2\pi} a^2 N v_T}{3}\left[A + B\frac{GM}{2a^2 v_T^2} + \frac{G^2 M^2}{2a^2 v_T^4}\Lambda\right]. \tag{7}$$

Here a is the reduced radius of the spherical masses. The first two terms in Eq. (7) describe the mechanism of absorption (falling on the center) of the light particles by the heavy ones in the model similar to Refs. [16,17]. The values of the constants A and B (order of units) depend on the specific mechanism of absorption (presence or absence of mass emission and the respective conditions for it [16]). The third term is the influence of gravitating scattering corresponding to Eq. (6). The effective frequency is calculated for a Maxwellian distribution of a heavy granular gas with the same (for simplicity) temperature T as the light component.

The cutoff in the logarithm Λ for the gravitating granular gas from physical reasons can be taken by using the impact parameters for two gravitating scattering bodies with the reduced radius a on the distances $r_{max} \approx n_0^{-1/3}$ and $r_{min} = \sqrt{(MmG/T)^2 + a^2}$. The Debye radius plays an important role in the stability of a gravitating system (see below), but evidently does not provide the screening of the gravity field, in contrast with the Coulomb potential in plasma. The maximal impact parameter therefore takes into account the limitation for the scattering due to the neighbors. The minimal impact parameter is connected with separation of scattering states and the falling down on the center.

The kinetic and Poisson equations (3) and (4) are to be linearized taking into account that linearization of the collision integral (5) yields

$$\delta I_{col} = -\nu\left(\delta f - \frac{f_0 \delta\rho}{m}\right), \tag{8}$$

as the density perturbation is expressed by means of the distribution function perturbation: $\delta\rho = m\int \delta f(\mathbf{r},\mathbf{v},t)\, d^3\mathbf{v}$. Then the standard procedure of linearization by using Laplace transformation results in the dispersion relation

$$\frac{\Omega^2}{k^2}\int \frac{[\mathbf{k}\cdot\partial f_0(\mathbf{v})/\partial\mathbf{v} + i\nu k^2 f_0(\mathbf{v})/\Omega^2]}{\omega + i\nu - \mathbf{k}\cdot\mathbf{v}} d^3\mathbf{v} = 1. \tag{9}$$

With $\nu = 0$ this equation, naturally, reduces to Eq. (5.27) in Binney and Tremaine [7]. Thus, collisions in a self-gravitating system, in addition to the trivial replacement $\omega \to \omega + i\nu$, result in a substantial (if k is not too small) modification of the spectrum. This modification is associated with the form (5) (of the collision integral) required in order to secure the particle conservation law.

Let us choose the x axis along $\mathbf{k}$ and assume that the initial velocity distribution is Maxwellian

$$f_0(\mathbf{v}) = \frac{1}{(2\pi)^{3/2} v_\mathbf{T}^3}\exp\left(-\frac{v^2}{2v_\mathbf{T}^2}\right). \tag{10}$$

Denote $v_x = u$ and integrate in Eq. (9) over v_y and v_z. We arrive at the dispersion relation in the following form:

$$\frac{\Omega^2}{k}\int_{-\infty}^{\infty}\left(\frac{df_0(u)}{du} + i\frac{\nu k f_0(u)}{\Omega^2}\right)\frac{du}{(\omega + i\nu - ku)} = 1, \tag{11}$$

where

$$f_0(u) = \int\int f_0(\mathbf{v}) dv_y dv_z = \frac{1}{\sqrt{2\pi} v_T}\exp\left(-\frac{u^2}{2v_T^2}\right) \tag{12}$$

and $\int_{-\infty}^{\infty} f_0(u) du = 1$.

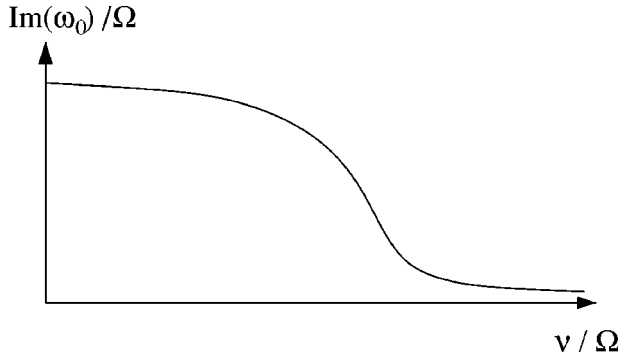

FIG. 1. Dimensionless increment of the Jeans instability $\mathrm{Im}(\omega_0)/\Omega$ for the value of wave vector $k=0$ plotted against the relation of the collision frequency ν to the value of the increment $\Omega \equiv \sqrt{4\pi G\rho_0}$ for $\nu=0$.

Equation (11) reduces to Eq. (13) in Landau [6] with $\nu=0$ if Ω^2 is replaced by $-\omega_0^2$ (ω_0 is the electron Langmuir frequency). By using expression (12) one obtains (Figs. 1 and 2) another form of the dispersion relation (11):

$$1+\frac{\Omega^2}{\sqrt{2\pi}(kv_T)^2}\int_{-\infty}^{\infty}\frac{\exp(-x^2/2)(x-i\nu kv_T/\Omega^2)}{\beta-x}dx=0, \tag{13}$$

where $x=u/v_T$, $\beta=(\omega+i\nu)/kv_T$.

Let us denote

$$J(\beta)=\frac{\beta}{\sqrt{2\pi}}\int_{-\infty}^{\infty}\frac{\exp(-x^2/2)}{\beta-x}dx. \tag{14}$$

Then, the dispersion relation (13) becomes

$$1-\frac{\Omega^2}{(kv_T)^2}\left[1-J(\beta)+\frac{i\nu(kv_T)^2}{\Omega^2(\omega+i\nu)}J(\beta)\right]=0. \tag{15}$$

The function $J(\beta)$ has the following asymptotic behavior (see, e.g., Ref. [18]):

$$J(\beta)\approx -i\sqrt{\frac{\pi}{2}}\beta, \quad \text{with} \quad |\beta|\ll 1, \tag{16}$$

and

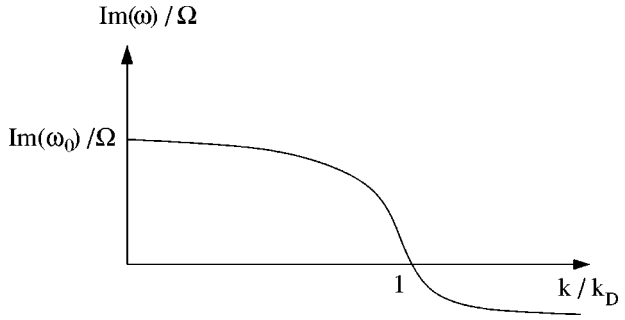

FIG. 2. Dimensionless increment of the Jeans instability $\mathrm{Im}(\omega_0)/\Omega$ plotted against the relation of the wave vector k to the Debye wave vector $k_D \equiv D^{-1}=\Omega/v_T$, where v_T is the thermal velocity of the small gravitating masses.

$$J(\beta)\approx -i\sqrt{2\pi}\beta\exp(-\beta^2/2), \quad \text{with}|\beta|\gg 1,$$
$$|\mathrm{Im}(\beta)|\gg|\mathrm{Re}(\beta)|,\ \mathrm{Im}(\beta)<0. \tag{17}$$

III. ANALYSIS OF THE DISPERSION RELATION

Let us consider an unstable mode, $\mathrm{Im}\,(\omega)>0$, for the case

$$|\omega+i\nu|\gg kv_T, \quad \text{that is } \beta\gg 1. \tag{18}$$

In this case we are able to integrate in Eq. (13) along the real axis, as there are no singularities: $x\neq\beta$. We expand $(1-x/\beta)^{-1}$ in Eq. (13) into a power series, taking into account symmetry and antisymmetry of the integrand. One obtains

$$1+\frac{\Omega^2}{(\omega+i\nu)^2}+\frac{3(kv_T\Omega)^2}{(\omega+i\nu)^4}-\frac{i\nu}{\omega+i\nu}-\frac{i\nu(kv_T)^2}{(\omega+i\nu)^3}=0. \tag{19}$$

With $\nu=0$ this is a quadratic equation for ω^2 with solution [compatible with the assumption (18)]:

$$\omega^2=3(kv_T)^2-\Omega^2, \tag{20}$$

which coincides with the Jeans dispersion equation (1), if $3v_T^2$ is replaced by $c_s^2=\gamma v_T^2$.

It is easy to take collisions into account in the limit of long wavelengths, $k\to 0$. Then, using Eq. (19) one obtains for an unstable mode $[\mathrm{Im}\,(\omega)>0]$

$$\omega(k=0)\equiv\omega_0=\frac{i}{2}(\sqrt{\nu^2+4\Omega^2}-\nu). \tag{21}$$

We arrive at the conclusion that the long-wavelength ($k\to 0$) mode remains unstable despite collisions which, naturally, diminish the instability growth rate.

An approximate solution of the dispersion relation (19) may be obtained by means of perturbation theory using the condition (18). Denote $\omega+i\nu=\tau=\tau_0+\tau_1$, $|\tau_1|\ll|\tau_0|$, where $\tau_0=\omega_0+i\nu$, and ω_0 is given by expression (21). Neglecting terms proportional to β^2 we find from Eq. (19) that τ_0 satisfies the following equation:

$$1+\frac{\Omega^2}{\tau_0^2}-\frac{i\nu}{\tau_0}=0, \tag{22}$$

and, hence,

$$\frac{\tau_1}{\Omega}=\frac{(kv_T)^2(3\Omega/\tau_0-i\nu/\Omega)}{2\Omega^2+\nu^2-i\nu\omega_0}. \tag{23}$$

From Eqs. (21) and (23) we arrive (after some algebra) at the following expression for $\omega=\omega_0+\tau_1$:

$$\omega=i\Omega\left[\sqrt{1+\frac{\nu^2}{4\Omega^2}}-\frac{\nu}{2\Omega}-\frac{3(k\,v_T)^2}{2\Omega^2}\varphi\left(\frac{\nu}{\Omega}\right)\right], \tag{24}$$

where

$$\varphi\left(\frac{\nu}{\Omega}\right)=\left[1+\frac{\nu^2}{4\Omega^2}+\frac{\nu\sqrt{\nu^2+4\Omega^2}}{4\Omega^2}\right]^{-1}\times\left[\frac{2\Omega}{\sqrt{\nu^2+4\Omega^2}+\nu}+\frac{\nu}{3\Omega}\right], \tag{25}$$

with the asymptotical behavior: $\varphi\to 1$ with $\nu/\Omega\to 0$; $\varphi=2\Omega/3\nu\to 0$ with $\nu/\Omega\gg 1$.

With $\nu=0$, naturally, one obtains

$$\omega=i\Omega\left(1-\frac{3(kv_T)^2}{2\Omega^2}\right), \tag{26}$$

which corresponds to the Jeans kinetic equation (20) if $(kv_T)^2\ll\Omega^2$.

As $\varphi=2\Omega/3\nu$ for $\nu/\Omega\gg 1$ one obtains from Eq. (24)

$$\omega=i\Omega\left(\frac{\Omega}{\nu}-\frac{(kv_T)^2}{\Omega\nu}\right). \tag{27}$$

Equation (27) shows that Im $(\omega)>0$. Thus, we are able to confirm our conclusion obtained above for a special case $k=0$: although collisions, naturally, diminish the instability growth rate, collision damping cannot quench the Jeans instability. This conclusion is obviously associated with the form (5) of the collision integral. If the τ approximation [which, in contrast to Eq. (5), does not conserve the number of particles] were used, then we would arrive at the linearized kinetic equation with a trivial replacement $\omega\to\omega+i\nu$ [this fact is also seen from the dispersion relation (9)]. In this case, with $kv_T\ll\Omega$ perturbations should be proportional to $\exp(-i\omega t)=\exp[(\Omega-\nu)t]$, so that with $\nu\geqslant\Omega$ the Jeans instability would be suppressed by collisions. Therefore the τ approximation cannot even give a qualitatively correct description for the Jeans long-wavelength mode.

Next we consider the case

$$|\beta|=\frac{|\omega+i\nu|}{kv_T}\ll 1. \tag{28}$$

In this case, we substitute the asymptotics (16) into the dispersion relation (15), which yields

$$\mathrm{Re}(\omega)=0,\quad \frac{\mathrm{Im}(\omega)}{kv_T}=\sqrt{\frac{2}{\pi}}\Psi\left(kD,\frac{\nu}{\Omega}\right), \tag{29}$$

where

$$D=\frac{v_T}{\Omega} \tag{30}$$

is an analog of the Debye radius in a gravitating system, and

$$\Psi=1-(kD)^2+\sqrt{\frac{\pi}{2}}\frac{\nu}{\Omega}\left(kD-\frac{1}{kD}\right). \tag{31}$$

Of course, the Debye-Jeans radius D is not associated with screening (as in the case of plasma); it is a characteristic scale of mass separation due to thermal motion. With $\nu/\Omega\ll 1$ Eq. (29) reduces to

$$\frac{\mathrm{Im}\ (\omega)}{kv_T}=\sqrt{\frac{2}{\pi}}[1-(kD)^2]\left(1-\sqrt{\frac{\pi}{2}}\frac{\nu}{\Omega}\right), \tag{32}$$

which holds both with $kD<1$ [when $\mathrm{Im}(\omega)>0$ and, hence, an instability arises] and with $kD>1$, when $\mathrm{Im}(\omega)<0$. If $\nu/\Omega\ll 1$ the condition $|kD-1|\ll 1$ has to be fulfilled in order to satisfy the inequality (28). Therefore, we put $kD=1$ in order to write the expression in the last brackets in Eq. (32) in a simplified form. Thus, with $kD>1$, $\nu/\Omega\ll 1$ a damping occurs (the term with collisions is a small correction). Formally, this damping for $\nu=0$ is reminiscent of collisionless Landau (1946) damping in plasma. But the mechanism of the damping is different. Landau damping in a plasma is associated with the inverse Cherenkov effect. This is not so in a gravitating system with $\mathrm{Re}(\omega)/k=0$, where the damping arises with $kD\geqslant 1$. We suggest the following explanation. A small sporadic displacement of a particle violates the balance of forces in the Jeans model of an infinite uniform self-gravitating gas at rest. As a result, the particle is to continue its motion in the same direction. This is the Jeans instability. Chaotic thermal motion tends to destroy this picture, and hence, to suppress the instability.

In order to satisfy the inequality (28) with $\nu/\Omega\gg 1$ the condition $kD\simeq[\sqrt{\pi/2}\ \ \nu/\Omega+\sqrt{2/\pi}\,\Omega/\nu]\gg 1$ is required. Then Eq. (29) leads to a strongly damped solution:

$$\frac{\mathrm{Im}(\omega)}{\Omega}=-\frac{\nu}{\Omega}-\frac{2}{\pi}\left(kD-\sqrt{\frac{\pi}{2}}\frac{\nu}{\Omega}\right). \tag{33}$$

According to expression (31), $\Psi=0$ with $kD=1$, so that $\omega=0$. It happens that this result does not depend on our assumption (28), which near the threshold $\omega=0$ becomes $\nu/\Omega\ll 1$ (as $kv_T\simeq\Omega$ with $kD\simeq 1$). Indeed, with $kD=1$, $\omega=0$ is the *exact* solution of the dispersion relation (15) with *any* value of the collision frequency ν.

It means we can find the k dependence for the spectrum near the value $\omega=0$ on the basis of the general dispersion relation (15) by expansion near the point $\omega=0$. In this way, with $\nu/\Omega\ll 1$ one obtains again the same expression (32), whereas for frequent collisions $\nu/\Omega\gg 1$ by using the additional assumption:

$$1\gg\frac{\nu}{\Omega}\exp\left(-\frac{\nu^2}{2\Omega^2}\right)\gg\frac{1}{\sqrt{2\pi}}(1-k^2D^2), \tag{34}$$

we arrive at the result:

$$\mathrm{Re}(\omega)=0,\frac{\mathrm{Im}(\omega)}{\Omega}=\sqrt{2\pi}(1-k^2D^2)\exp\left(\frac{\nu^2}{2\Omega^2}\right). \tag{35}$$

Inequalities (34) are easily fulfilled with $kD\simeq 1$ and $\nu\gg\Omega$. In the point $kD=1$ the unstable regime transforms into a stable one.

Thus, we arrive at the following conclusion: although collisions result in a substantial modification of the spectrum diminishing the instability growth rate, they do not affect the Jeans instability criterion: $kD<1$. The threshold $\omega=0$ (separating stable and unstable modes) corresponds to $kD=1$ with *any* value of ν.

Now we consider a case

$$|\beta| = \frac{|\omega + i\nu|}{kv_T} \gg 1, \quad |\mathrm{Im}(\beta)| \gg |\mathrm{Re}(\beta)|, \quad \mathrm{Im}(\beta) < 0. \tag{36}$$

For the asymptotics (36) the general dispersion relation (15) possesses a few solutions. At first we consider the solution with $\mathrm{Re}(\beta) \equiv \mathrm{Re}(\omega)/(kv_T) = 0$. In this case the dispersion relation (15) reduces to

$$\left(|\mathrm{Im}(\beta)| + \frac{\nu}{kv_T}k^2D^2\right)\exp\left(\frac{[\mathrm{Im}(\beta)]^2}{2}\right) = \frac{1}{\sqrt{2\pi}}(k^2D^2 - 1). \tag{37}$$

It is obvious that the solution of Eq. (37) exists only if $kD \gg 1$. If, in addition, $|\mathrm{Im}(\beta)| \gg \nu/kv_T$ one obtains

$$\mathrm{Re}(\omega) = 0, \frac{\mathrm{Im}(\omega)}{\Omega} = -\sqrt{2}kD\sqrt{\ln\frac{(kD)^2}{\sqrt{2\pi}}} - \frac{\nu}{\Omega}. \tag{38}$$

In fact, the last term on the right-hand side of Eq. (38) is small and can be omitted.

If $|\mathrm{Im}(\beta)| \ll \nu k^2D^2/kv_T = \nu kv_T/\Omega^2$ we find from Eq. (37):

$$\mathrm{Re}(\omega) \equiv 0, \quad \mathrm{Im}(\omega) = -\sqrt{2}kv_T\sqrt{\ln\left(\frac{kv_T}{\sqrt{2\pi}\nu}\right)}. \tag{39}$$

This damping mode exists only with $\ln(kv_T/\nu) \ll \nu^2(kD)^4/(kv_T)^2$.

Finally, for $\mathrm{Re}(\beta) \neq 0$ the general dispersion Eq. (15) also has the solutions with $|\mathrm{Im}(\beta)| \gg |\mathrm{Re}(\beta)|$. In the case under consideration Eq. (37) for $\mathrm{Im}(\beta)$ happens to be also valid, as it follows from the general dispersion (15). The small (in comparison with the imaginary part) real part $\mathrm{Re}(\beta)$ is determined from the expansion of Eq. (15):

$$\mathrm{Re}(\beta) = \frac{2\pi s}{|\mathrm{Im}(\beta)|}, \tag{40}$$

where $s \ll |\mathrm{Im}(\beta)|^2$ is an integer.

IV. CONCLUSION

The kinetic treatment of the Jeans instability shows that collisions may result in substantial modification of the oscillation spectrum. In the long-wavelength limit, $\mathbf{k} \to 0$, an aperiodic unstable mode was found. According to Eq. (21), with $\nu \gg \Omega$, the instability growth rate is $\mathrm{Im}(\omega) = \Omega^2/\nu$, whereas with $\nu \ll \Omega$ it reduces to the value $\mathrm{Im}(\omega) = \Omega$ given by the Jeans equation (1).

With small but finite k values, the mode of oscillation remains aperiodic: $\mathrm{Re}(\omega) = 0$, the instability growth rate [given by Eq. (24)] with $\nu \gg \Omega$ strongly depends on the collision frequency ν. Collision damping (although diminishing the growth rate) cannot quench the Jeans instability. This conclusion is associated with the form (5) of the collision integral. If the so-called τ approximation [which, in contrast to Eq. (5), does not conserve the number of particles] were used, perturbations would be proportional to $\exp[(\Omega - \nu)t]$, so that with $\nu \geqslant \Omega$ the Jeans instability would be suppressed by collisions. It is necessary to mention that in the recent paper [19] the Jeans instability for the gravitating dusty plasmas with charged and neutral grains has been considered in framework of hydrodynamical approximation. It was shown that collisions between neutral and charged grains cannot totally quench the Jeans instability for small values of k and propagating modes in the region of existence are strongly damped. Our results on the basis of kinetic consideration for two-component system of neutral gravitating grains corroborate these statements, although the kinetic consideration leads of cause to some different from the hydrodynamic concrete expressions and dependencies for the increments and decrements of the modes.

The threshold $\omega = 0$ (separating stable and unstable modes) was found not to depend on the collision frequency ν. It is determined only by the Debye-Jeans radius D of a gravitating system: $\omega = 0$ if $kD = 1$ with *any* ν value. Thus, we arrived at the conclusion that collisions do not affect the Jeans instability criterion: $kD < 1$. However, even in the neighborhood of the threshold $\omega = 0$ (i.e., $kD \sim 1$) the spectrum of oscillations strongly depends on the collision frequency ν: according to Eq. (35), this dependence is exponential. Propagating (rather than aperiodic) modes $\mathrm{Re}(\omega)/k \neq 0$ are also found [see Eqs. (37)–(40)]. These modes, however, are strongly damped as $|\mathrm{Re}(\omega)| \ll |\mathrm{Im}(\omega)|$.

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